Set Theory
Events

Let $B$ denote the event that the number that shows is two or smaller. Then,

- $B = \{2\}$
- $B = \{\{1\}, \{2\}\}$.
- $B = \{1, 2\}$.

What’s the difference between 2 and 3?
Set Complement

Recall that $E$ is the event that the number that shows is even. $O$ is the event that the number that shows is odd. Let $E^c$ denote the complement of the set $E$. Then,

- $EE^c = \{1\}$.
- $E \cup E^c = \{2, 3, 4, 6\}$.
- $E^c$ is the event that the number that shows is not two.
- $E^c = O$.  

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Muually exclusive sets

Which statement is true about mutually exclusive events and partition of \( \Omega \)?

- \( A \) and \( A^c \) are mutually exclusive, and they form a partition of \( \Omega \).

- If \( E_1, E_2 \) and \( E_3 \) are mutually exclusive, then \( E_1E_2 = \emptyset \), and \( E_1E_3 = E_1 \).

- If \( E_1, E_2 \) and \( E_3 \) form a partition of \( \Omega \), then \( E_1 \cup E_2 = \emptyset \).

- If \( E_1, E_2 \) and \( E_3 \) are mutually exclusive, then they form a partition of \( \Omega \).
Inclusion-Exclusion

Property p.8  \[ P(A \cup B) = P(A) + P(B) - P(AB). \] That is because, \( A \cup B \) can be written as the union of three mutually exclusive sets: \[ A \cup B = (AB^c) \cup (A^c B) \cup (AB). \] So

\[
P(A \cup B) = P(AB^c) + P(A^c B) + P(AB)
\]
\[
= (P(AB^c) + P(AB)) + (P(A^c B) + P(AB)) - P(AB)
\]
\[
= P(A) + P(B) - P(AB).
\]

Property p.9  \[ P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC). \] This is a generalization of Property p.8 and can be proved in a similar way.
Basic probability
We roll a fair die and flip a fair coin. What is the probability that the number that shows on the die is in the set $\{1, 3\}$ and the coin shows head?
Basic probability

We roll a fair die. What is the probability that the number that shows is strictly bigger than 3 or odd?
Mutually exclusive

Let $A, B$ and $C$ be mutually exclusive events with $P(A) = P(B) = P(C) = 0.1$. What is the probability of $P(A \cup B \cup C)$?
Counting
Principle of Counting

Three people take one ball at random without replacement from a bag containing 6 balls with distinct colors. How many possible outcomes are there for the three drawn balls by the three people? What would be the answer if the balls are drawn one at a time by a person with replacement, i.e. each person draws a ball at random and put it back in the bag?
Over-Counting

What is the number of six letter sequences that can be obtained by
ordering letters of the set \{A, A, B, B, B, C\}? For example, \textit{ACBBAB}
is a possibility. We don’t distinguish the A’s from each other and we don’t
distinguish the B’s from each other.
N choose K

How many binary sequences of length 6 have (exactly) 4 or 3 ones? Two such sequences are 110110 and 111000.
N choose K

How many ways are there to go from the left lower corner of the following table to the right upper corner of the table assuming that you can go one step up or one step to the right on the lines at each move (not being able to move down or to the left)?
**N choose K**

How many ways are there to order the letters of FOOTBALL such that the three letters OOA are consecutive? FOAOTBLL and FOOATBLL are two possibilities.
Ordering?

Suppose there are eight socks in a bag, numbered one through eight, which can be grouped into four pairs: \( \{1, 2\}, \{3, 4\}, \{5, 6\}, \) or \( \{7, 8\} \). The socks of each pair have the same color; different pairs have different colors. Suppose there are four (distinct!) people present, and one at a time, they each draw two socks out of the bag, without replacement. Suppose all socks feel the same, so when two socks are drawn from the bag, all possibilities have equal probability. Let \( M \) be the event that each person draws a matching pair of socks. What is \( P(M) \)? Present your answer in an irreducible fraction.
Poker Challenge
### Problem Solving

**Table 1: Poker Challenge**

<table>
<thead>
<tr>
<th>Poker hand</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Royal flush</td>
<td>$1.5 \times 10^{-6}$</td>
</tr>
<tr>
<td>Straight flush</td>
<td>$1.5 \times 10^{-5}$</td>
</tr>
<tr>
<td>Four of a kind (quads)</td>
<td>$2.4 \times 10^{-4}$</td>
</tr>
<tr>
<td>Full house</td>
<td>0.0014</td>
</tr>
<tr>
<td>Flush</td>
<td>0.00198</td>
</tr>
<tr>
<td>Straight</td>
<td>0.0039</td>
</tr>
<tr>
<td>Three of a kind</td>
<td>0.0211</td>
</tr>
<tr>
<td>Two pairs</td>
<td>0.0475</td>
</tr>
<tr>
<td>One pair</td>
<td>0.423</td>
</tr>
<tr>
<td>else</td>
<td>0.501</td>
</tr>
</tbody>
</table>
Problem Solving

Poker

Suppose a deck of playing cards has 52 cards, represented by the set $C$:

$$C = \{1C, 2C, \ldots, 13C, 1D, 2D, \ldots, 13D, 1H, 2H, \ldots, 13H, 1S, 2S, \ldots, 13S\}.$$  

Here $C, D, H, \text{ or } S$ stands for the suit of a card: "clubs," "diamonds," "hearts," or "spades."

Suppose five cards are drawn from a standard 52 card deck of playing cards with all possibilities being equally likely.
Flush

*FLUSH* is the event that all five cards have the same suit. What is $P(\text{FLUSH})$?
Two Pair

*TWO PAIR* is the event that two cards both have one number, two other cards both have some other number, and the fifth card has a number different from the other two numbers. What is $P(TWO\ PAIR)$?
Three of a kind

THREE OF A KIND is the event that three of the cards all have the same number, and the other cards have numbers different from each other and different from the three with the same number. What is $P(THREE\ OF\ A\ KIND)$?
Four of a kind

*FOUR OF A KIND* is the event that four of the five cards have the same number. What is $P(\text{FOUR OF A KIND})$?
Play with Binomial Coefficients
Identity 1

\[ 2^n = \sum_{k=0}^{n} \binom{n}{k} \]
Identity 2

\[
\binom{n}{n-k} = \binom{n}{k}
\]

for \(0 \leq k \leq n\).
Identity 3

\[
\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}
\]

for \(1 \leq k \leq n\).
Identity 4

\[
\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2.
\]
Questions?